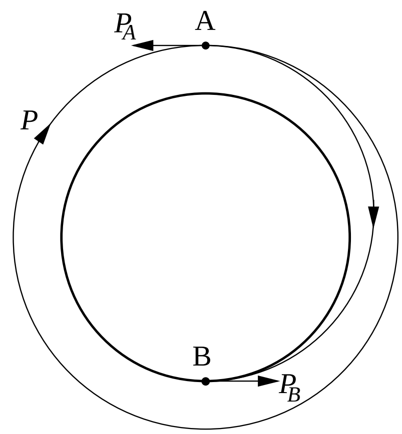
In each section, qu. 1 is revision of relevant bookwork, qu. 2 is short and/or straightforward, and qu. 3 is a longer problem. The last part of qu. 4.3 is a more challenging problem for enthusiasts.

1. Orbits

1.1 Write down the relevant conservation laws for a point mass moving under the influence of a central, conservative force. Use them to derive a differential equation for the radial motion.

1.2 Discuss the stability and closure of almost-circular orbits as a function of n for a central force $F \propto r^n$. [Discuss at least the cases $n = 1$, $n = -1$, and $n = -2$; and address more general n too, for example, $n = -6$.

1.3 A lunar excursion module is initially in a circular orbit at a height $R/4$ above the lunar surface, where R is the radius of the moon. The objective is to land at point B by firing the module's rockets briefly at points A and B as indicated. Find the required impulses P_A and P_B, in terms of the initial momentum P of the module. (Ignore the rotation of the moon.)



[*Ans:* $P_A = 0.057P$, $P_B = 1.179P$.]

2. Rigid body dynamics

2.1 Define the *inertia tensor*, *principal axes* and *principal moments of inertia* of a rigid body, and explain their relevance to the angular velocity and angular momentum of the body.

2.2 Recall that a *spherical top* is a rigid body for which all the principal moments of inertia are equal. Show that a uniform cone of mass M with height h equal to the diameter of its base is a spherical top with moment of inertia $I = 3Mh^2/40$.

2.3 Such a cone rolls freely without slipping on a horizontal table, with its curved surface in contact with the table. Show that this is only possible if the angular velocity ω of the cone about its axis satisfies

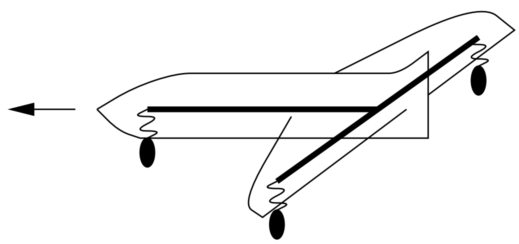
$$\omega^2 < \frac{20\sqrt{5}}{3}\frac{g}{h} .$$

What happens when this condition is violated?

3.1 Explain what is meant by the *normal modes* of oscillation of a many-particle system, and how their frequencies can be found.

3.2 Discuss why the specific heats of gases at moderately high temperatures are in the sequence $H_2 < O_2 < H_2O < CO_2$.

3.3 An aircraft taking off is (crudely!) represented by two identical thin rods joined rigidly in a T configuration, with landing wheels attached to the ends by identical springs, as illustrated below.



(i) Show that the normal mode frequencies ω are given by

$$\omega^2 = 12\frac{k}{M}\,, \quad \frac{24}{5}\left(1 \pm \frac{1}{\sqrt{6}}\right)\frac{k}{M}$$

where k is the spring constant of each spring and M is the mass of the aircraft.

(ii) Describe the oscillations excited when (a) the front wheel, (b) a side wheel passes over a bump of height h in the runway. Assume that $\omega\tau \ll 1$, where τ is the time taken to go over the bump.

4. Elasticity

4.1 Define the *bending moment* B and the *moment of area* I for a bent beam. Derive the relation $B = YI/R$ where Y is Young's modulus and R is the radius of curvature.

4.2 A uniform steel ruler of width a and thickness b is clamped at its lower end in a vertical position with a length l protruding above the clamp. A small sideways force F is applied at the upper end. Find the displacement y as a function of the height x above the clamp. [*Ans:* $y = 2Fx^2(3l - x)/(Yab^3)$.]

4.3 (i) Show more generally that when a distributed transverse force $f(x)$ per unit length is applied to a beam the equilibrium displacement, when small, satisfies the differential equation

$$YI\frac{d^4y}{dx^4} = f(x)\,.$$

(ii) Hence show that free transverse oscillations of the ruler in qu. 4.2 satisfy the differential equation

$$\frac{Yb^2}{12\rho}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0$$

where ρ is the density.

(iii) Show that the possible angular frequencies of transverse oscillation of the ruler are of the form

$$\omega = \frac{\alpha^2 b}{2l^2}\sqrt{\frac{Y}{3\rho}}$$

where α is a solution of the equation $\cosh\alpha = -\sec\alpha$. (The smallest is $\alpha = 1.87\ldots$)